The quaternion group

The <u>quaternion group</u> is defined

$$Q_8 = \{1, -1, i, -i, j, -j, k, -k\}$$

w/ 1 as the identity, and the products computed

$(-1)\cdot a = a(-1) = -a \quad \forall \ a \in Q_8$

$i \cdot i = j \cdot j = k \cdot k = -1$

$i \cdot j = k, \ j \cdot k = i, \ k \cdot i = j, \ j \cdot i = -k, \ k \cdot j = -i, \ i \cdot k = -j.$

<u>Fact</u>: Q_8 is a group: 1 is the identity,

and $(i)(-i) = (i)(i)(-1) = (-1)(-1) = 1$

and similarly $(-i)(i) = 1$, so $i^{-1} = -i$, $j^{-1} = -j$, $k^{-1} = -k$

$(-1)(-1) = 1$, so $(-1)^{-1} = -1$. So every element has an inverse.

Associativity holds but is tedious to check.

<u>Notice</u>: Q_8 is not abelian. Thus it is not cyclic.

However, we can write all elements in terms of i, j

$1 = i^0$	$j = j^1$
$-1 = i^2$	$-j = j^{-1}$
$i = i^1$	$k = ij$
$-i = i^{-1}$	$-k = ji$

Thus $\{i, j\}$ is a generating set for Q_8.

On HW: Is $D_8 \cong Q_8$?